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# Ballistic electron spectroscopy

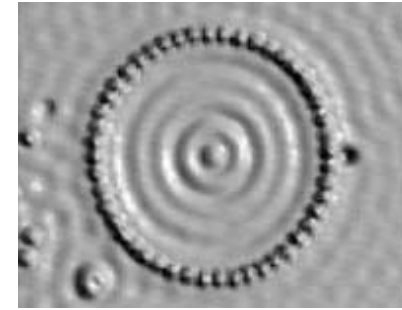
*Frank Hohls*

- I. Introduction – low dimensional electron systems and ballistic electrons
- II. The idea – ballistic electron spectroscopy
- III. Charge readout of detector
- IV. Some data – proof of concept

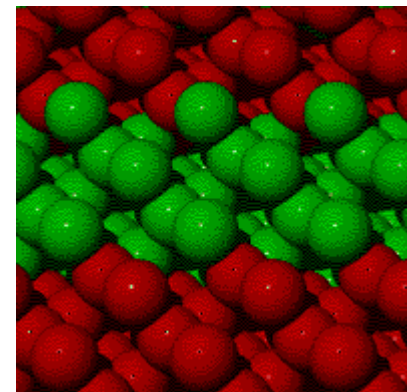
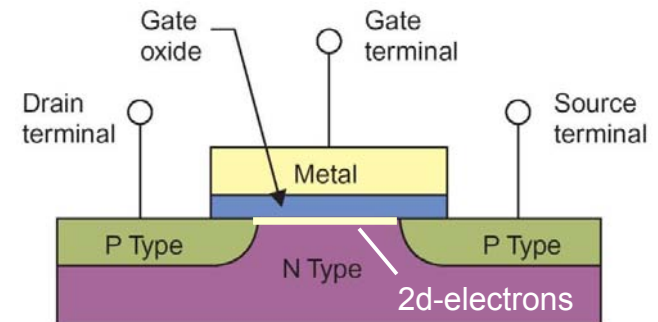
# 2d electrons – where do I find them?

## Confining electrons to an interface

- ◆ Electrons in metal: thin metal films or surface
  - high electron density, short length scales (Fermi length, mean free path)
  - ultra high vacuum
- ◆ Electrons in semiconductor: Field effect transistor (Silicon-MOSFET)
  - + easy tunable electron density
  - ◆ Quantum Hall Effect (QHE)  
Nobel prize 1985
- ◆ Heterostructure grown atomic layer by layer (Nobel prize 2000)
  - + Very clean system
  - ◆ Fractional QHE, Nobel prize 1998
  - ◆ ballistic electron effects

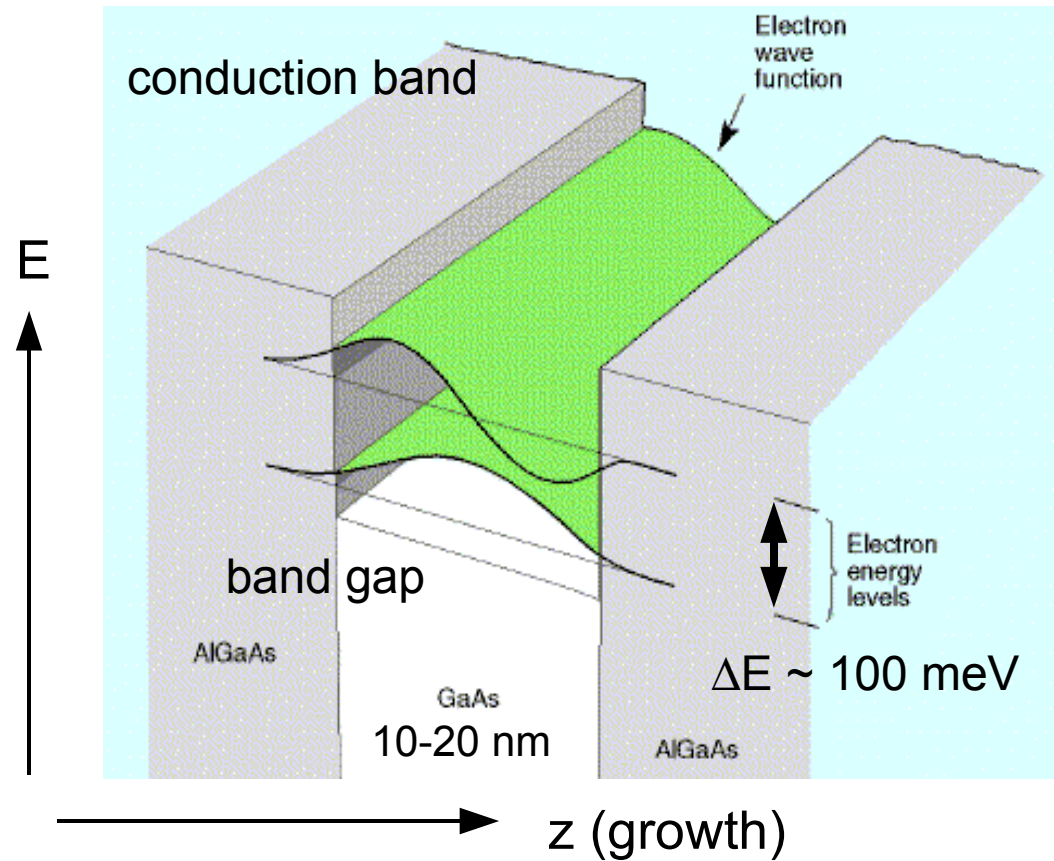


standing waves on metal surface



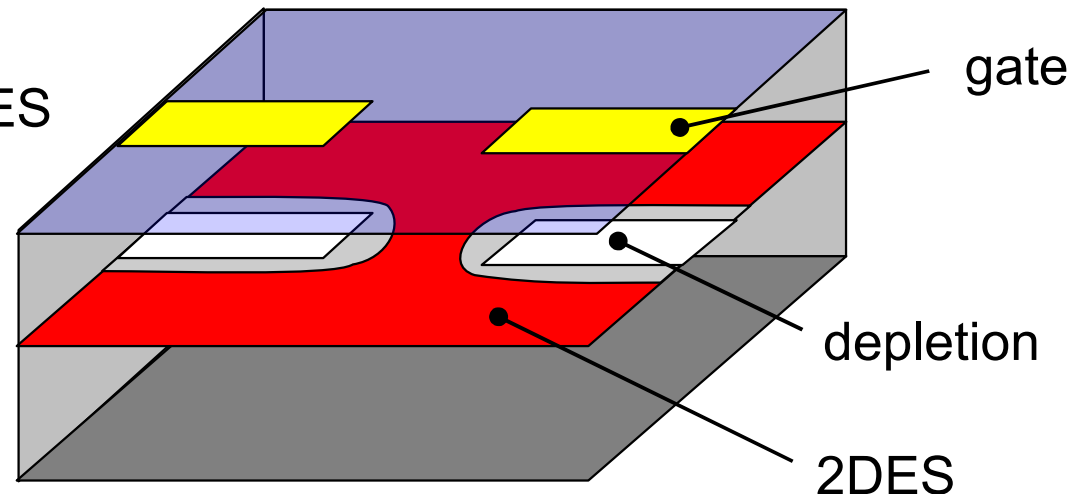
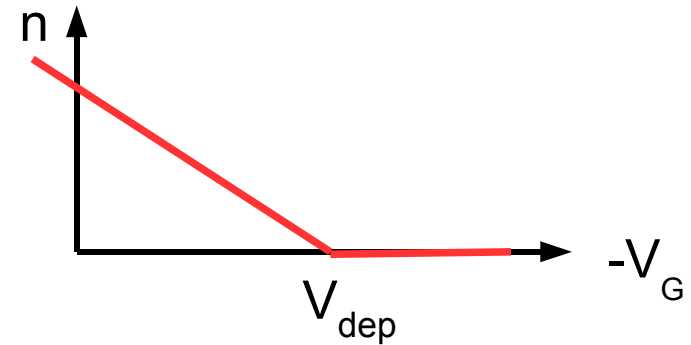
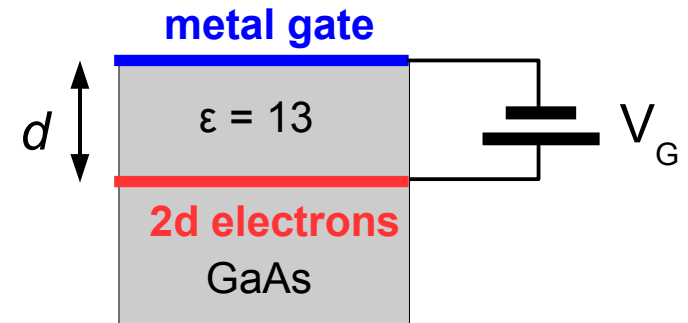
# 2d electrons in a heterostructure

- ◆ Engineering of band edge to create a quantum well
  - ◆ Quantized energy in growth direction ( $z$ )
  - ◆ Free motion in  $x$ - $y$  plane
- ◆ Electron density  $n \sim 10^{15} \text{ m}^{-2}$ 
  - ◆ Electron distance  $\sim 30 \text{ nm}$
  - ◆ Fermi wave length  $\sim 80 \text{ nm}$
- ◆ High interface quality and remote doping
  - ◆ Very high mobility  $\mu$   
(measured from  $\sigma = en\mu$ )
  - ◆ Mean free path of up to  $100 \mu\text{m}$



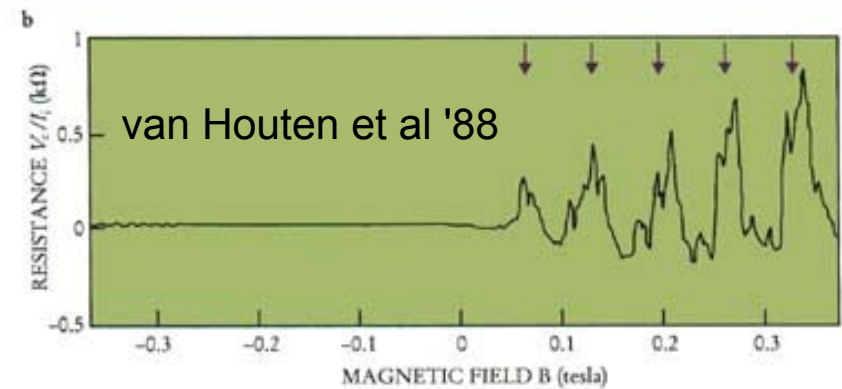
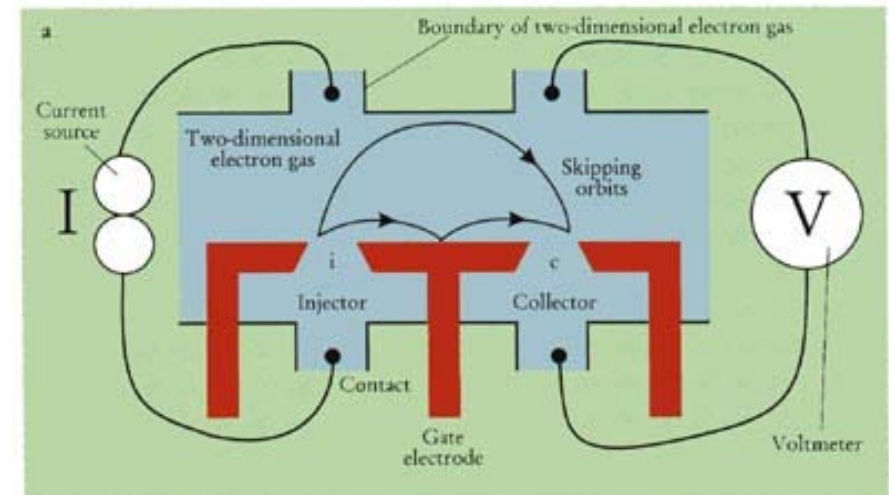
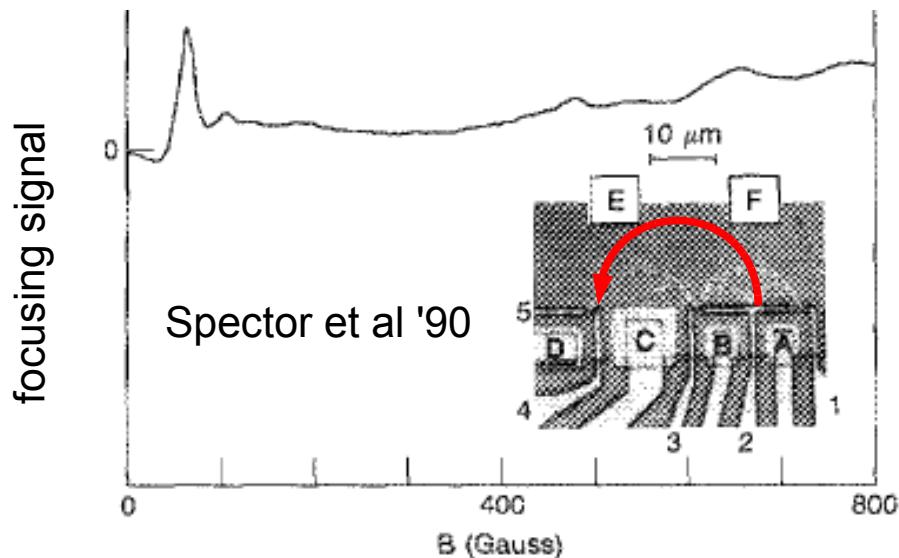
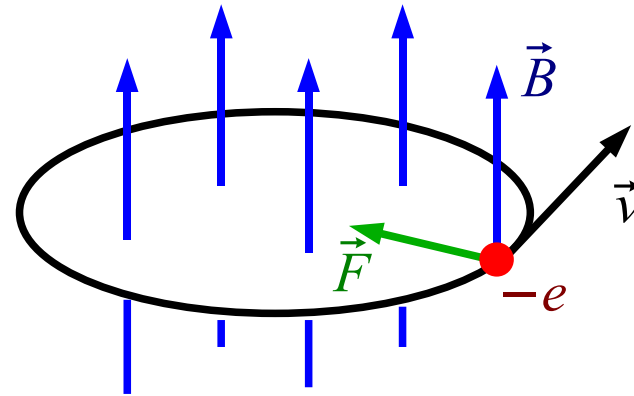
# Getting smaller – gating the 2d electrons

- ◆ Metal on GaAs forms Schottky Diode
  - ◆ Current  $I \sim \exp(eV_G/kT)$  negligible for  $V_G < 0$  and low  $T$
- ◆ Gate and 2d electron sheet form plate capacitor
  - ◆  $C = \epsilon\epsilon_0 \cdot A/d$ ,  $\Delta q = C \cdot V_G = A \cdot e \cdot \Delta n$
  - ◆ Change of electron density  $n$  by applied voltage
  - ◆ 1 gate + 2 contacts = MESFET
- ◆ Depletion of electrons beyond threshold voltage  $V_{\text{dep}}$ 
  - ◆ Transfer gate pattern into 2DES



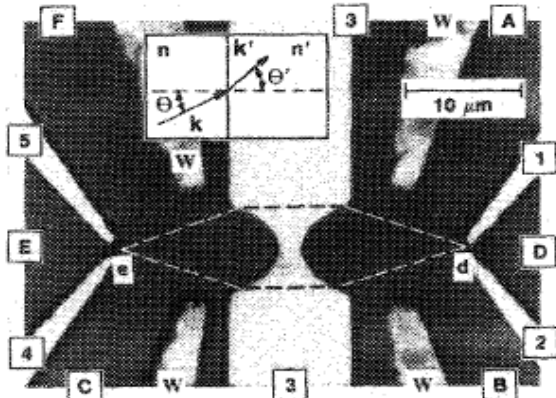
# Ballistic electrons in 2d – focusing

- ◆ Electrons travel large distance without scattering
  - ◆ ballistic transport
- ◆ Revealed in magnetic focusing experiments
  - ◆ Lorentz force  $\sim B \cdot v_F$  in perpendicular magnetic field  $B$
  - ◆ Electrons forced on cyclotron orbits in with  $r_C \sim 1/B$



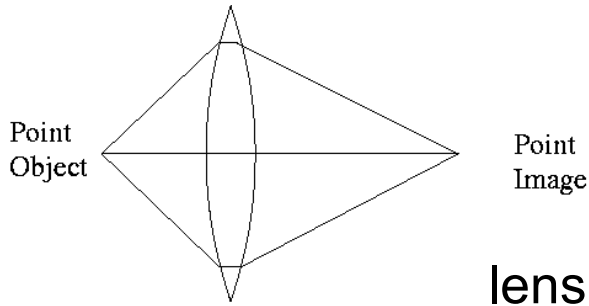


# Ballistic electrons – optics and interference



Spector et al '90

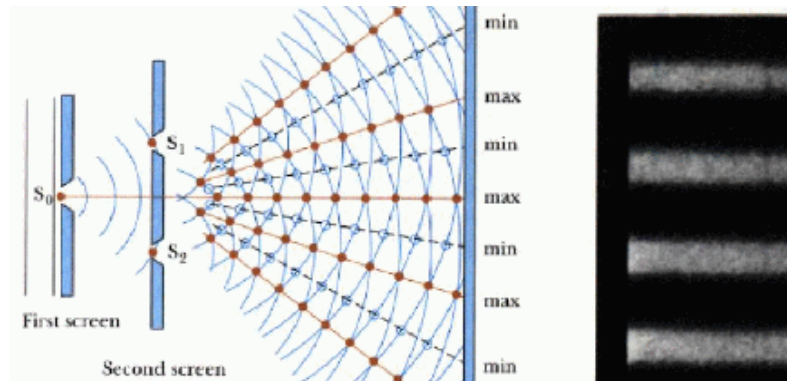
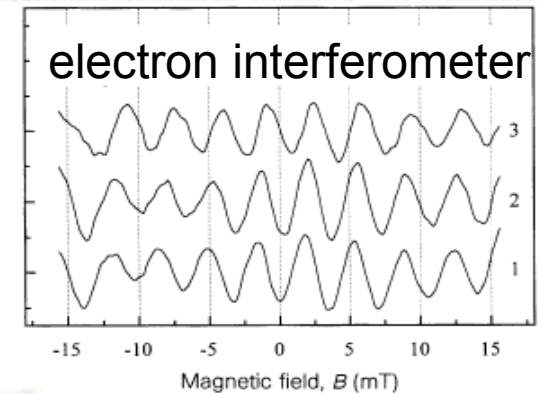
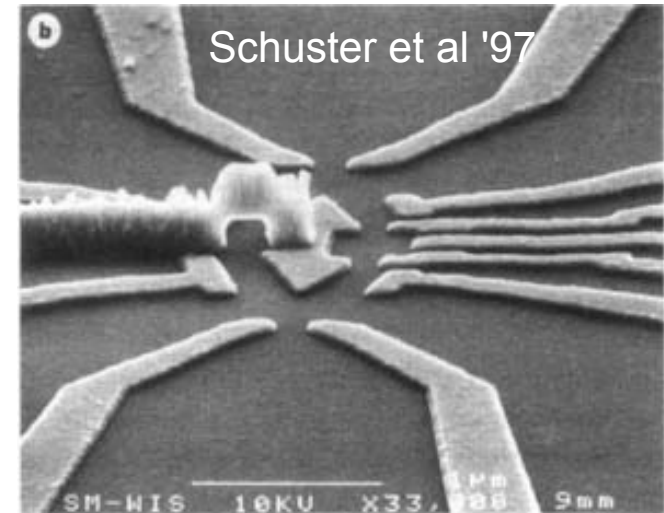
Electron lens



lens

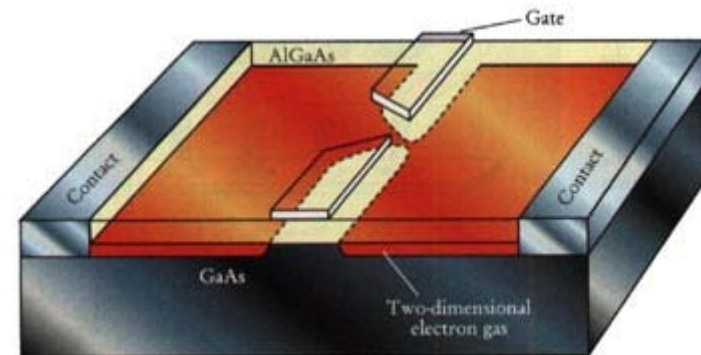
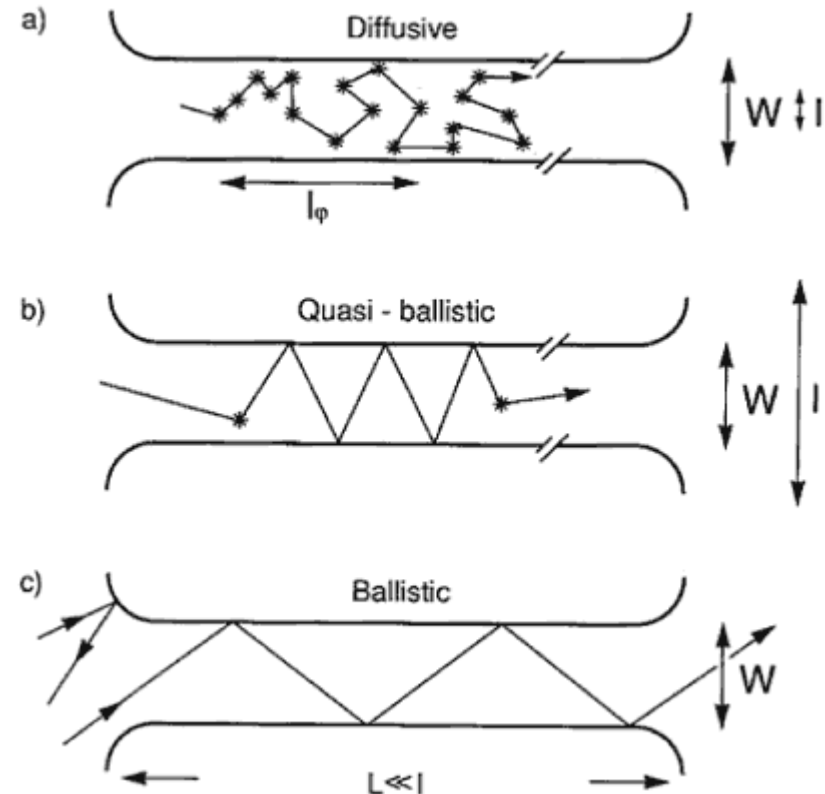
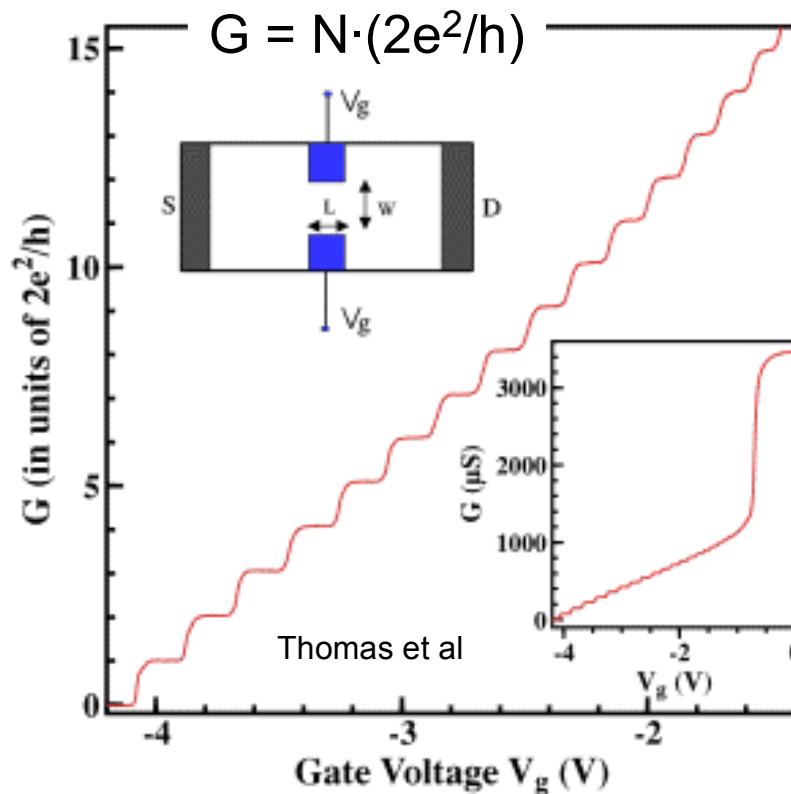
double slit

- ◆ Fermi wave vector  $k_F \sim \sqrt{n}$  can be changed by gate
- ◆ Electron optics
- ◆ Wave nature of electrons
- ◆ interference effects
- ◆ Double slit interferometer with magnetic field
- ◆ Aharonov-Bohm effect  $\Delta\varphi \sim \Phi/\Phi_0$  ( $\Phi$  flux)
- ◆ Analogues to light optics



# Ballistic 1d wire – quantized conductance

- ◆ Ballistic wire: width  $w$  and length  $L$  much smaller than scattering length  $l$ 
  - ◆ no backscattering within channel
  - ◆ quantized conductance (Wharam et al '88, van Wees et al '88)



# 1d wire – why quantized in $2e^2/h$ ?

- ◆ Quantized energy perpendicular to wire (y)
- ◆ Free motion in 1d along wire (x)
- ◆ For one 1d sub-band:

- ◆ Density of states 1d (no spin):

$$D(E) = \sqrt{\frac{m}{2h^2 E}} = \frac{1}{hv}$$

- ◆ Current

$$I = e \cdot v \cdot [D(E) \cdot \Delta E] = \frac{e^2}{h} V$$

- ◆ Add spin degeneracy  $g = 2$

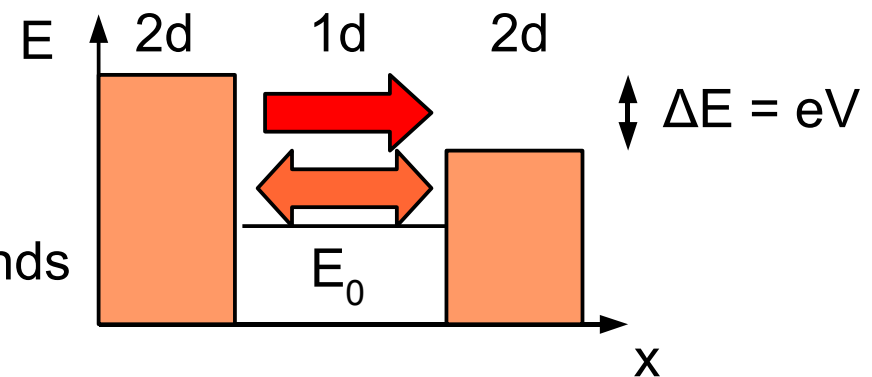
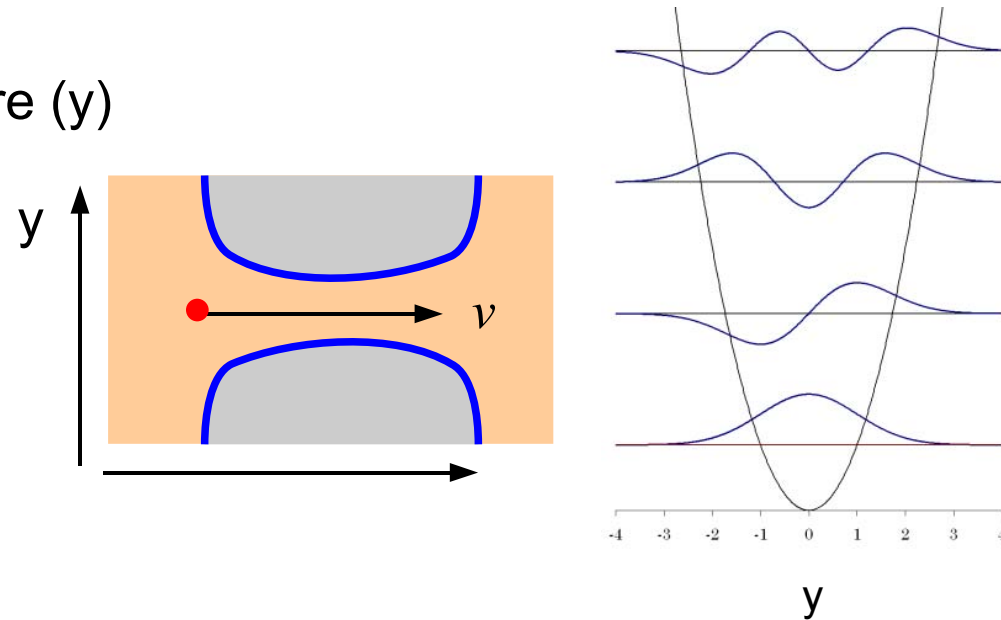
$$G = \frac{I}{V} = \frac{2e^2}{h}$$

- ◆ Add contributions of N occupied 1d sub-bands

- ◆  $G = N (2e^2/h)$

- ◆ Conductance steps smoothed for short wire

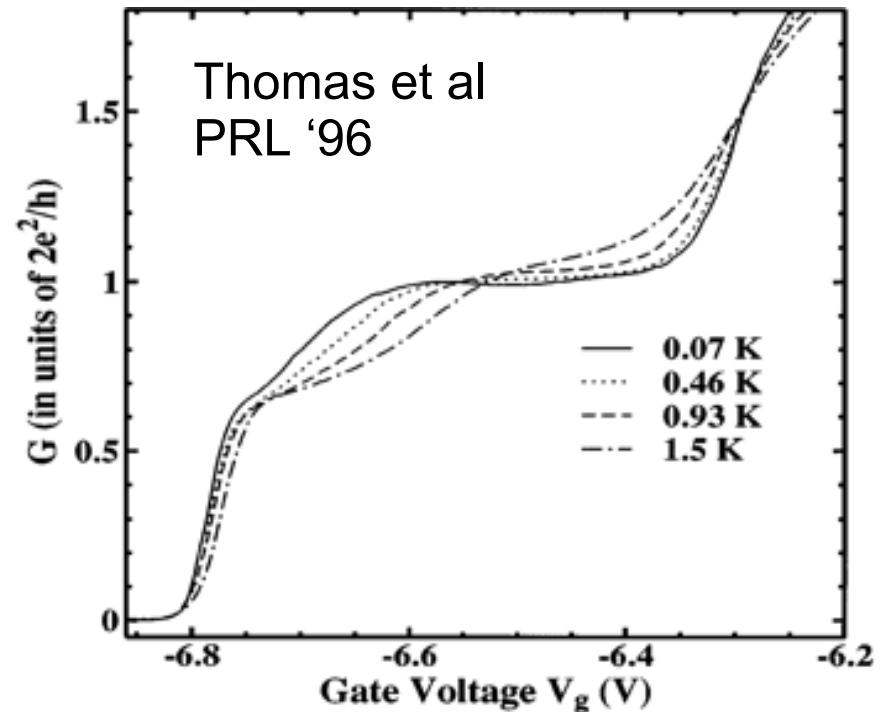
- ◆ Quantum point contact (QPC)



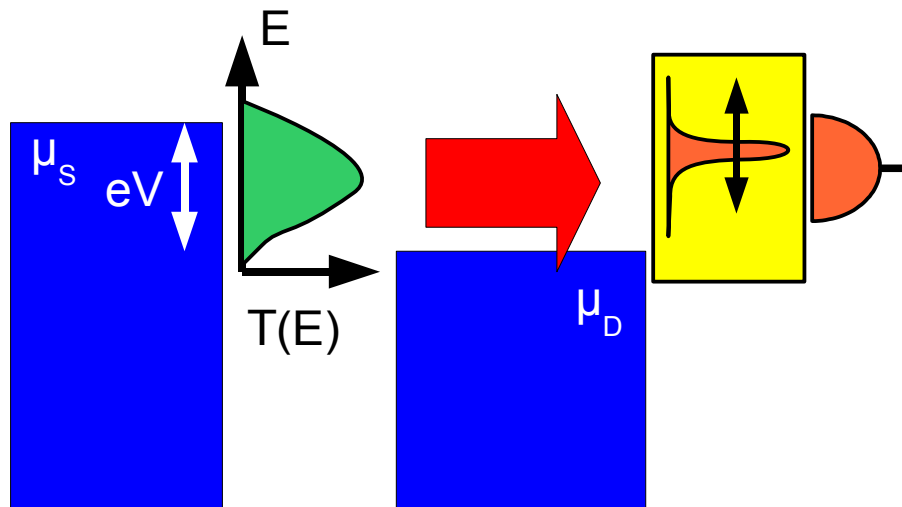
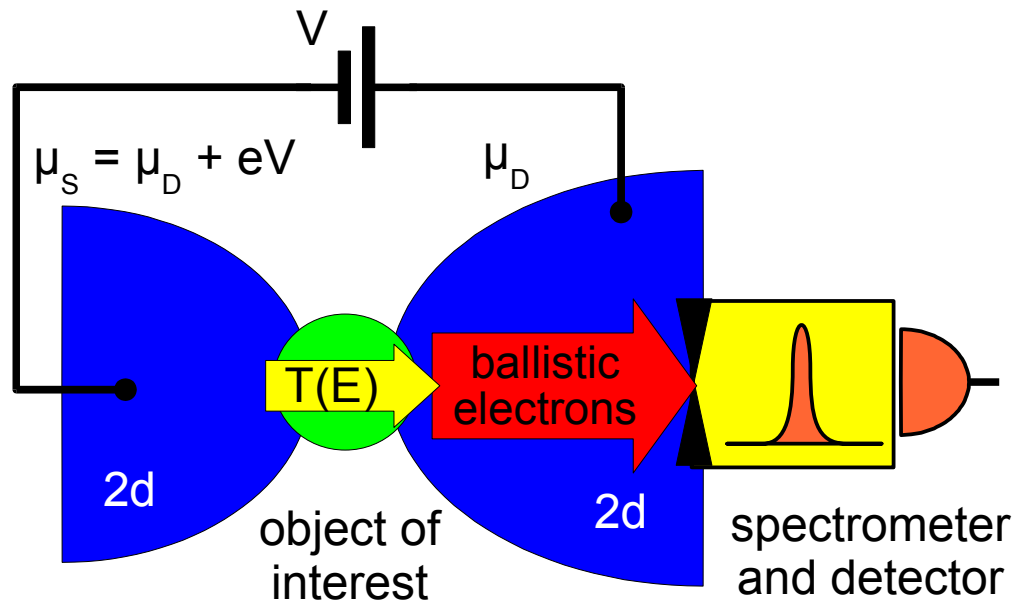


# Open questions: Electron interaction in 1d

- ◆ Additional conductance plateau at  $G = 0.7 \cdot (2e^2/h)$
- ◆ Effect of **electron-electron interaction**
- ◆ Different models proposed
  - ◆ Kondo model
  - ◆ Spontaneous spin polarization
  - ◆ Jumping of subbands
- ◆ For all models **we expect change of density of states  $D(E)$**
- ◆ **How to measure** without changing the system (no back-action) ?



# Proposal: ballistic electron spectroscopy



- ◆ Source **non-equilibrium ballistic electrons** by an object of interest
  - ◆ Energy distribution maps Transmission  $T(E)$
  - ◆ **Inner structure mirrored by transmission**
- ◆ **Energy resolved detection of ballistic electrons**
- ◆ No back-action of measurement on object of interest
- ◆ Use **quantum dot** as spectrometer  
APL 89, 212103 (06)

# Quantum dot – electrons in 0d

- ◆ Confinement in all dimensions

- ◆ Quantum dot (QD)

- ◆ N electrons in QD

- ◆ 1<sup>st</sup> approximation: disk with capacitance C

- ◆ Coulomb blockade energy:  
 $E_C = q^2/C = e^2/C$

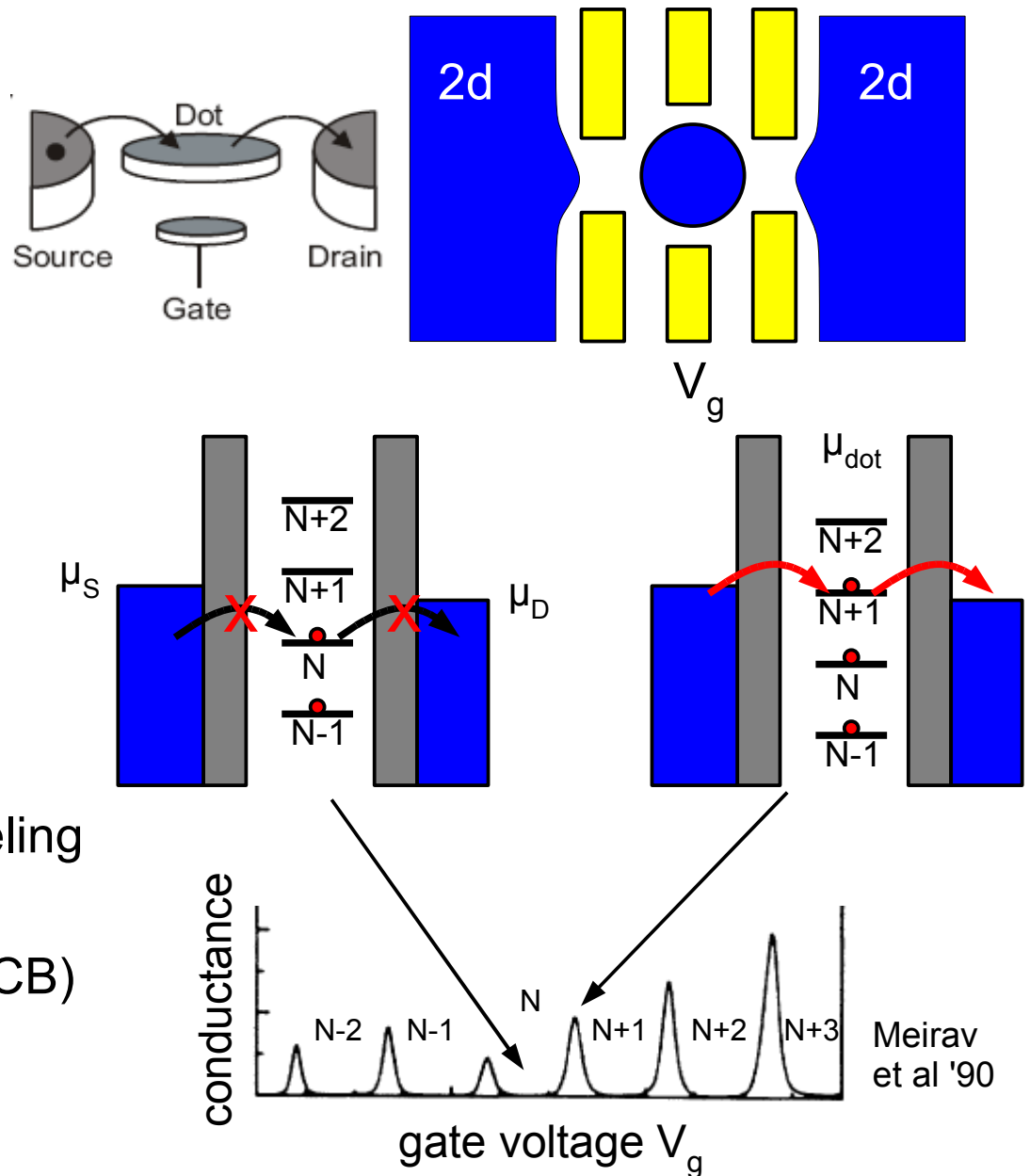
- ◆ Spacing of dot chemical potential:  
 $\mu_{N+1} - \mu_N = E_C$

- ◆ Shift potential of quantum dot with gate  $V_g$

- ◆ Resonant single electron tunneling only for  $\mu_S > \mu_{N+1} > \mu_D$

- ◆ otherwise Coulomb blockade (CB)

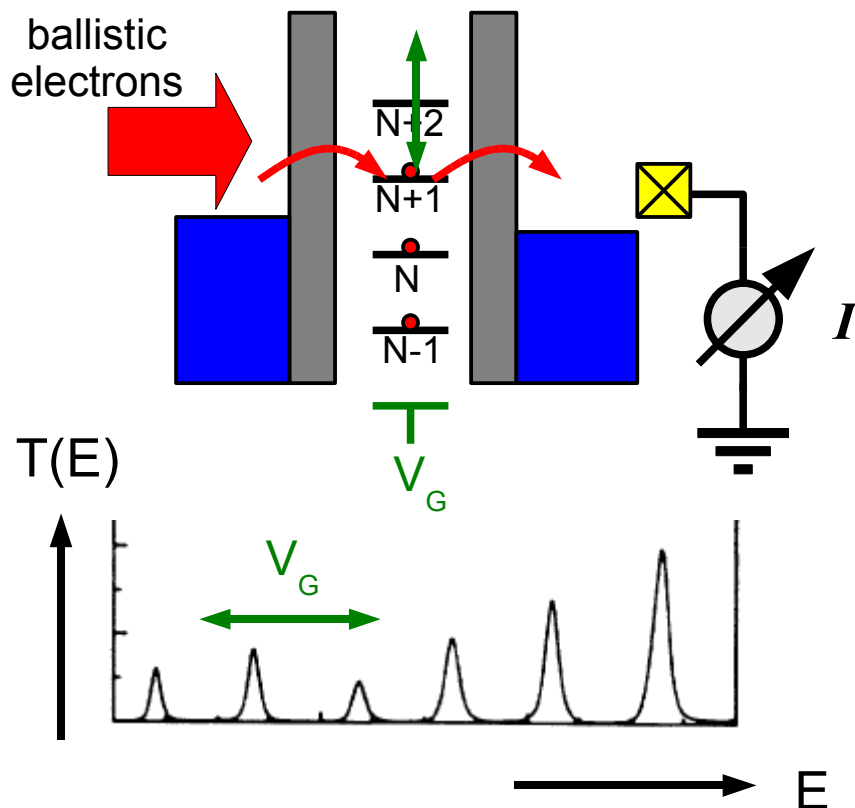
- ◆ Conductance G displays peak for each change in electron number



# Analogue: Fabry-Perot spectrometer

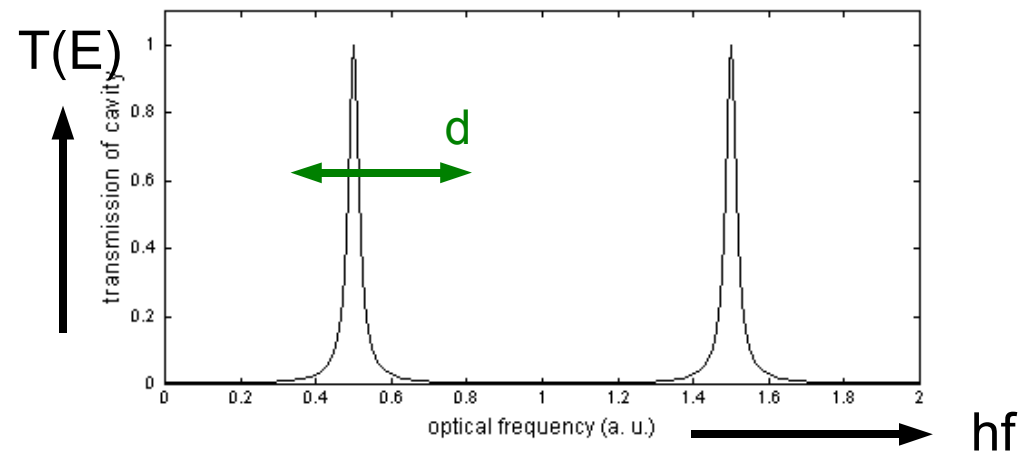
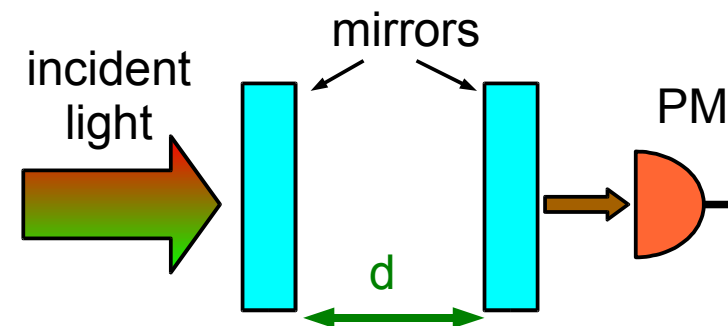
## Quantum dot spectrometer

- ◆ Transmission for selected energies (frequencies)
- ◆ Periodic transmission
- ◆ Energy comb shifted by applied gate voltage



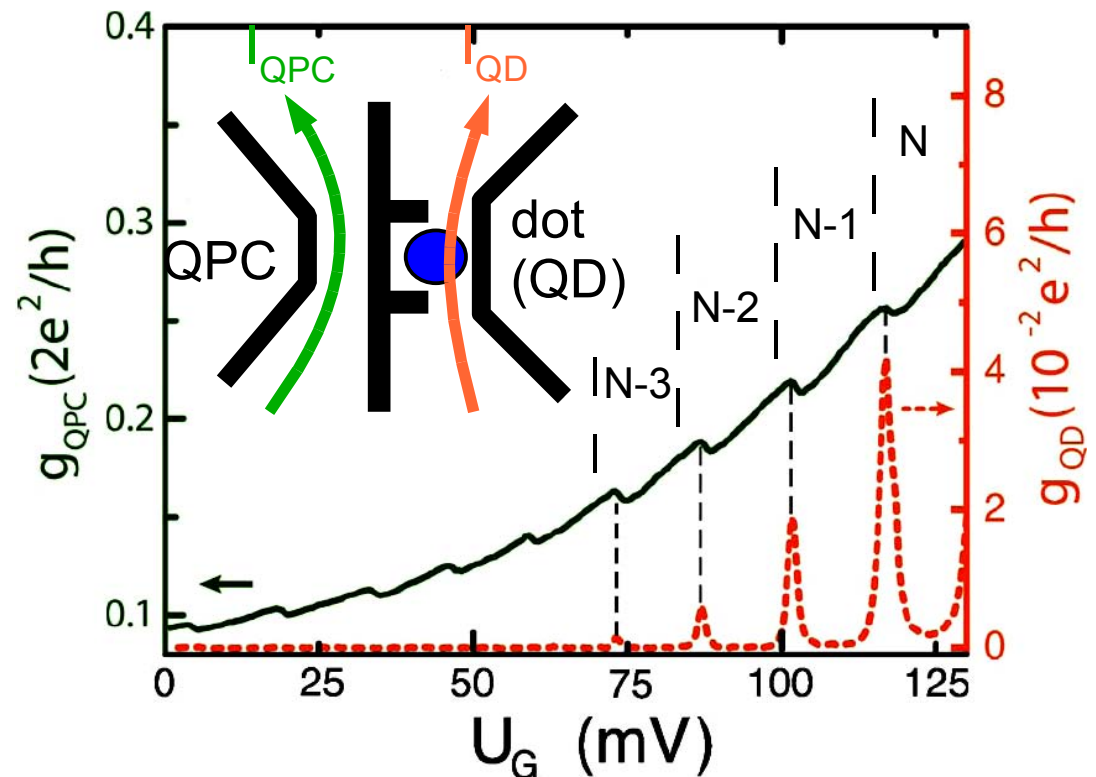
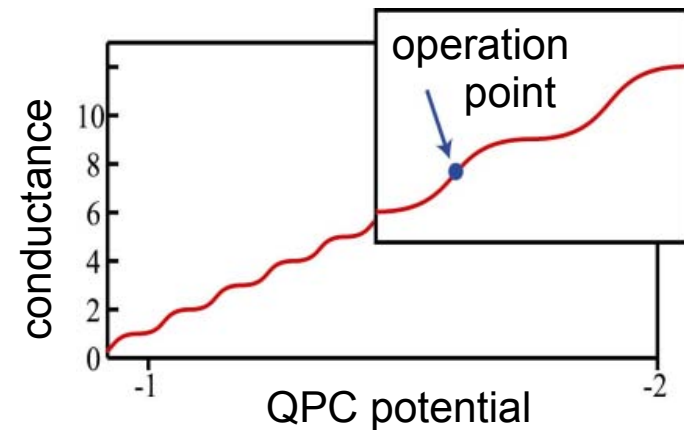
## Fabry Perot spectrometer

- ◆ Frequency comb tuned by distance  $d$



# Quantum dot charge detection

- ◆ Detect charge state of quantum dot
- ◆ Use 1d wire near dot (Field et al '93)
  - ◆ Potential in wire changes due to charge on quantum dot
  - ◆ At step edge conductance very sensitive to local potential
- ◆ Step in wire conductance for each change of quantum dot charge



C. Fricke *et al.*, PRB '05

M. Rogge *et al.*, PRB '05

# Spectrometer with charge readout

- ◆ Occupation of the dot depends on occupation of impinging electrons  $\Omega_{\rightarrow}(E)$  and accessible empty outgoing states  $1 - \Omega_{\leftarrow}(E)$  ( $0 \leq \Omega \leq 1$ )

$$\Gamma_{0 \rightarrow 1} = 2\gamma \Omega_{\rightarrow}(E) \quad \Gamma_{1 \rightarrow 0} = \gamma(1 - \Omega_{\leftarrow}(E))$$

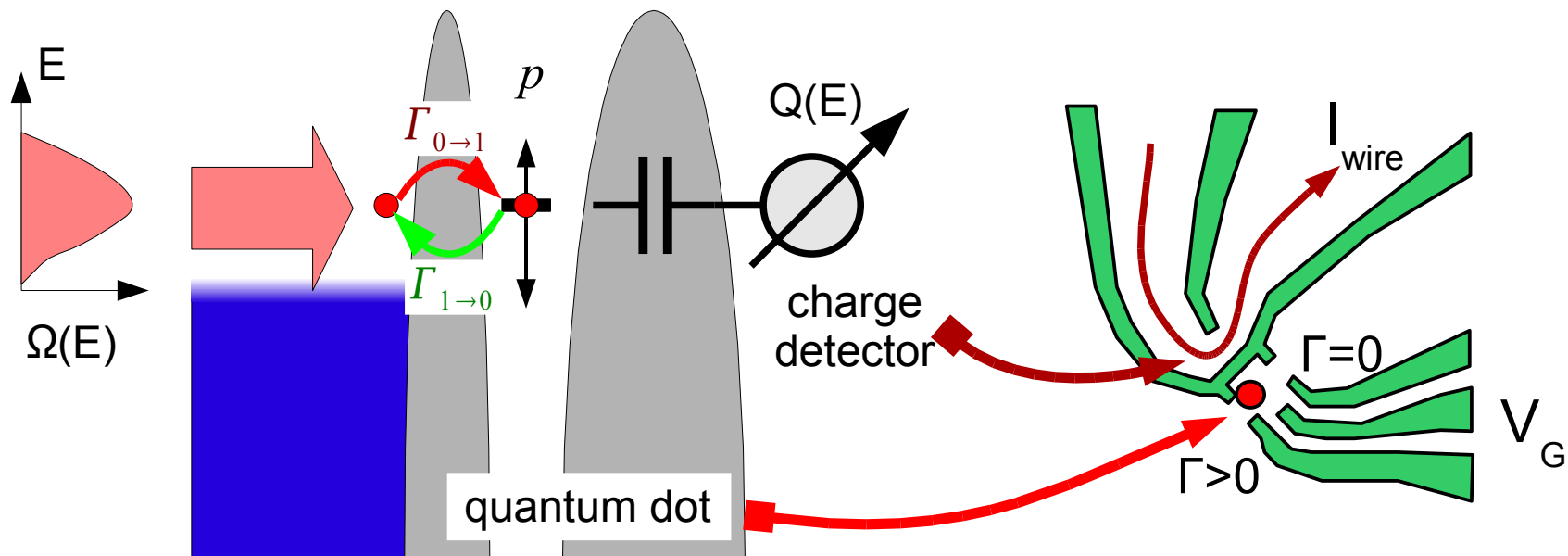
- ◆ Dot occupancy  $p$  ( $0 \leq p \leq 1$ ):

$$(1 - p)\Gamma_{0 \rightarrow 1} = p\Gamma_{1 \rightarrow 0}$$

- ◆ Measure charge  $Q = -pe$

- ◆ For dilute ballistic electrons:

$$\left. \begin{array}{l} \Omega_{\rightarrow} \ll 1 \\ (1 - \Omega_{\leftarrow}) \approx 1 \end{array} \right\} \Rightarrow p \approx 2\Omega_{\rightarrow}$$



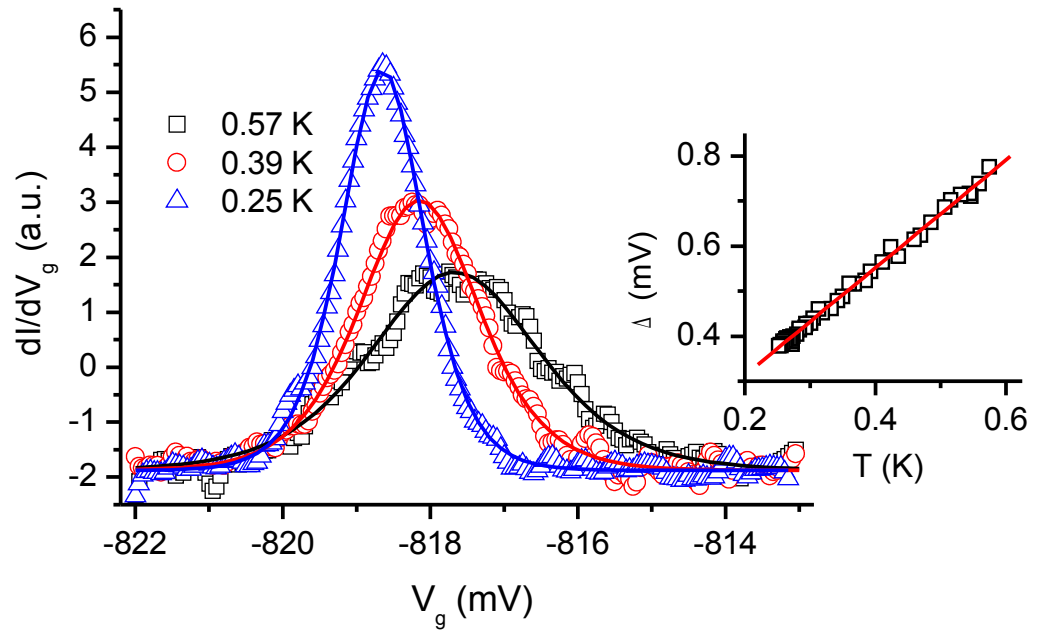
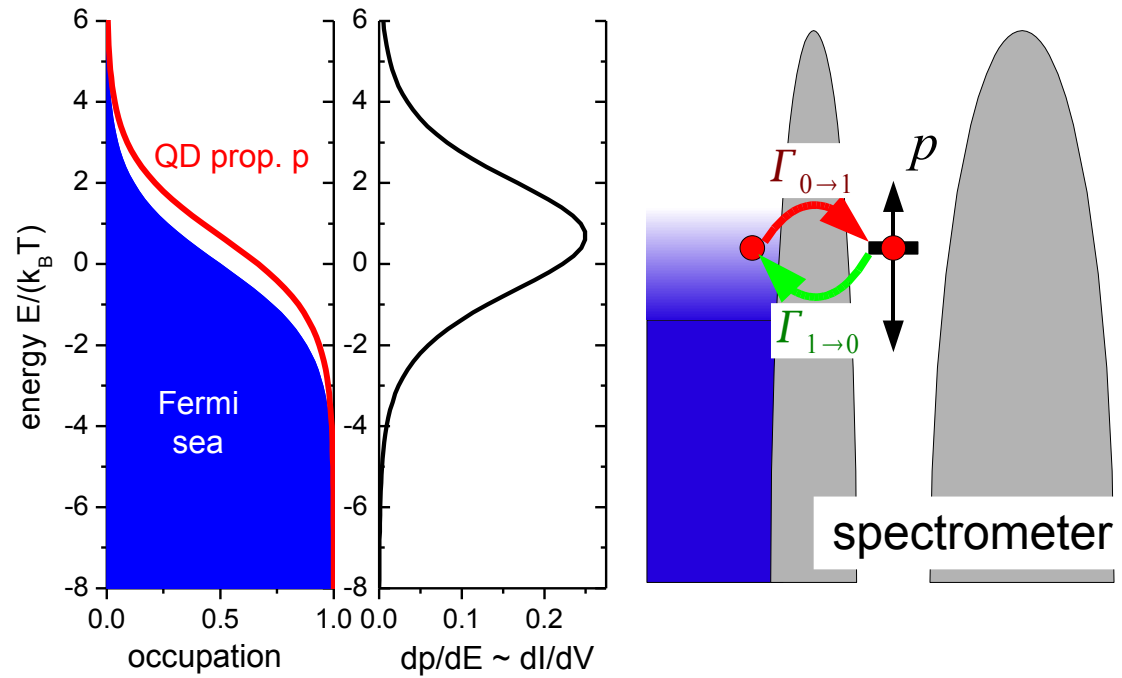


# Spectrometer calibration

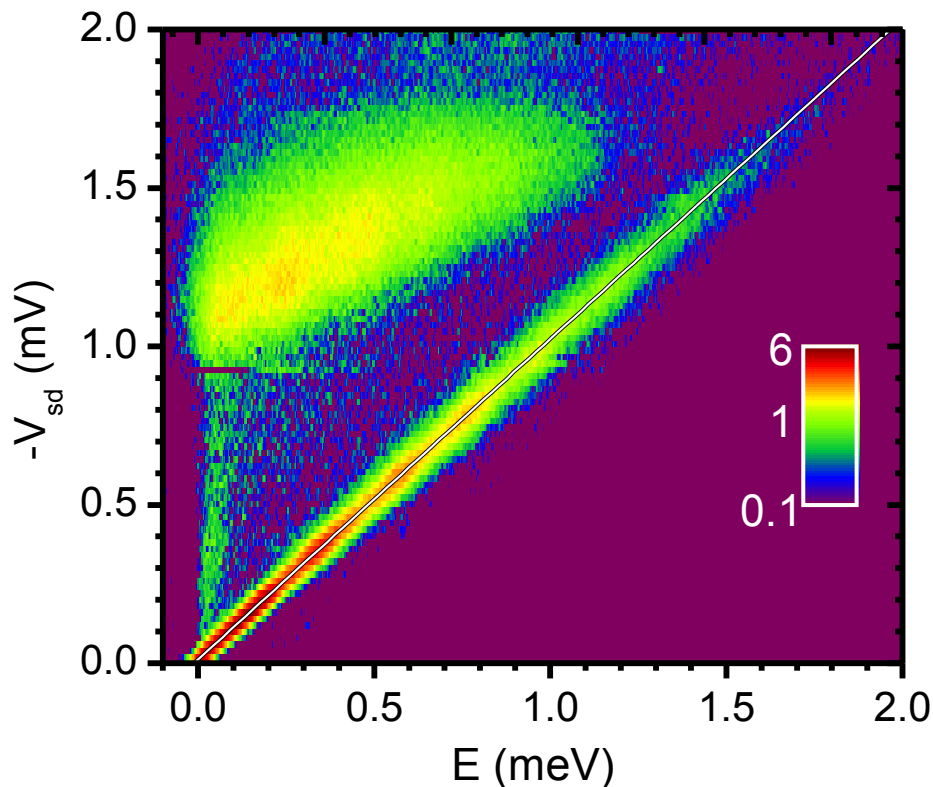
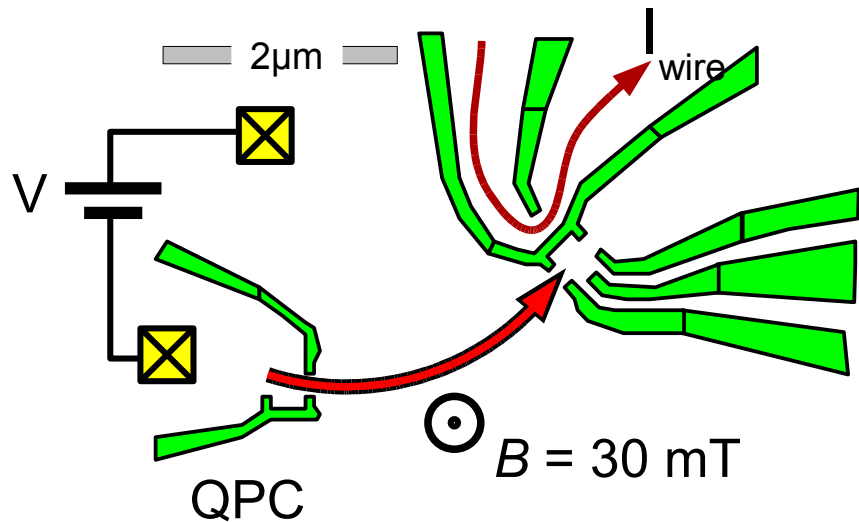
- ◆ Spectrometer signal near Fermi energy maps Fermi distribution  $f(E)$

$$p = \frac{2}{1 + \frac{1}{f(E)}}$$

- ◆ Measure  $dI/dV_g \sim dp/dE$
- ◆ Use temperature dependence of peak width  $\Delta$  for spectrometer calibration
- ◆  $\Delta E/\Delta V_g = 0.0719 \pm 0.0007$

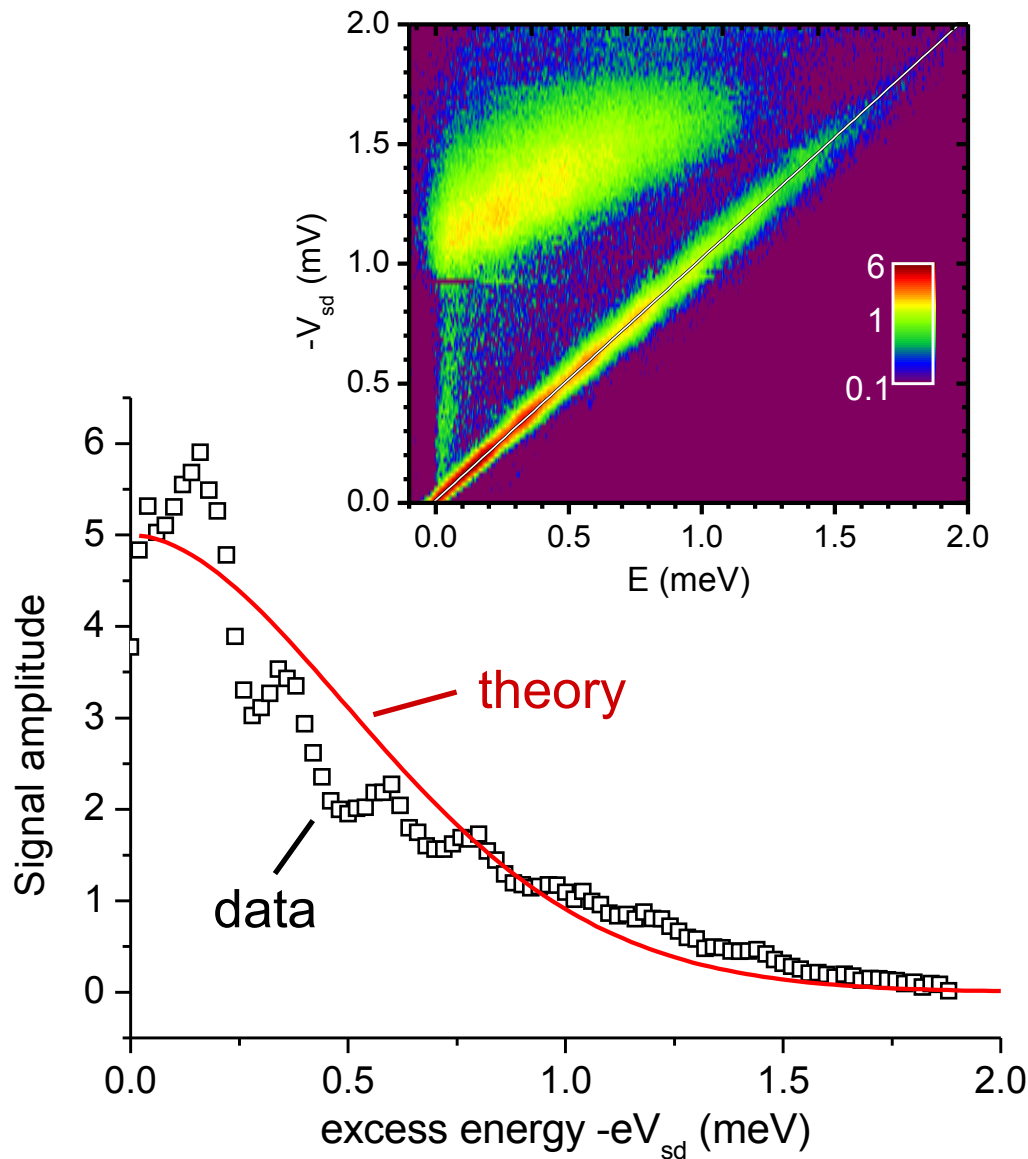


# Ballistic signal

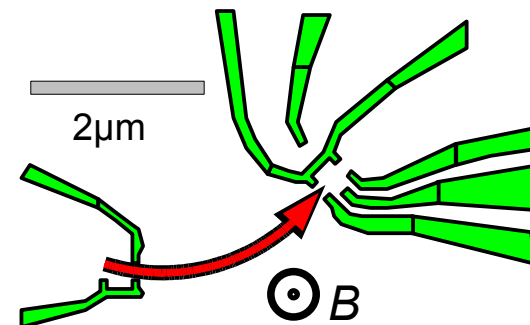


- ◆ Source ballistic electrons with QPC set to first plateau for well defined injection spectrum
  - ◆ Inject  $\Omega(E) = \text{const}$
- ◆ Magnetic field  $B$  bends ballistic electrons into the spectrometer
- ◆ Vary maximum energy  $E_{\text{max}} = -eV_{sd}$  of ballistic electrons
- ◆ AC-modulation: Mark electrons injected at  $E_{\text{max}}$
- ◆ Linear shift of ballistic peak position:  $-eV_{sd} = 1.01 \cdot E$ 
  - ◆ No energy scaling factor  $E = -\alpha \cdot eV_{sd}$  with  $\alpha < 1$  as claimed previously
- ◆ It works!

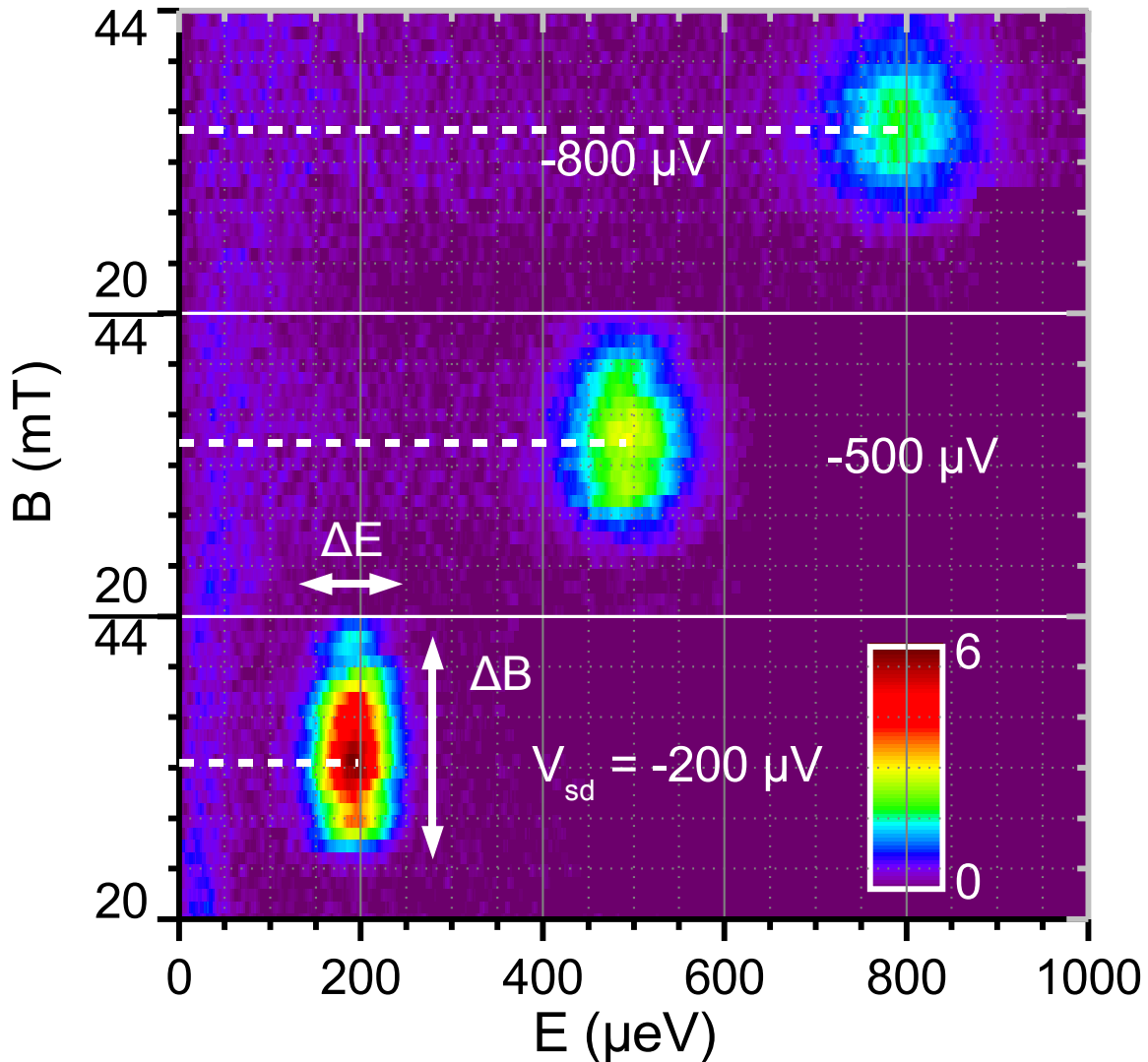
# Ballistic peak – amplitude



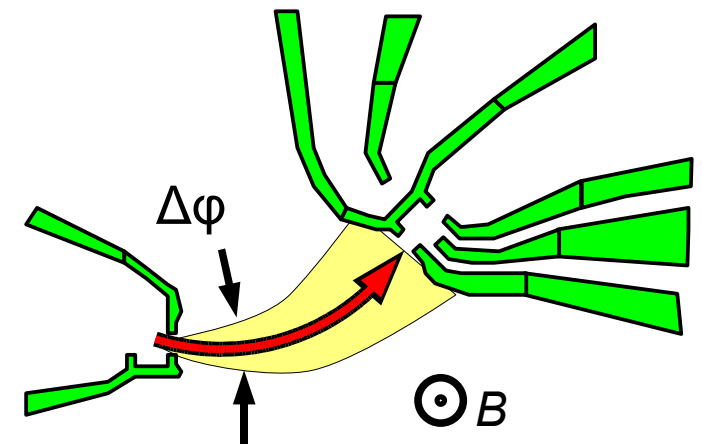
- ◆ Signal decays for rising energy
  - ◆ Scattering of ballistic electrons
- ◆ Compare to **theory for e-e scattering** with equilibrium electrons (Giuliani and Quinn, 1982)
  - ◆ Deviation due to large population of non-equilibrium electrons?
  - ◆ Agreement observed in interference experiment (Yacoby *et al* '91) – difference between phase and energy relaxation?
- ◆ Superimposed oscillations due to **interference** of different paths



# Energy + angle distribution

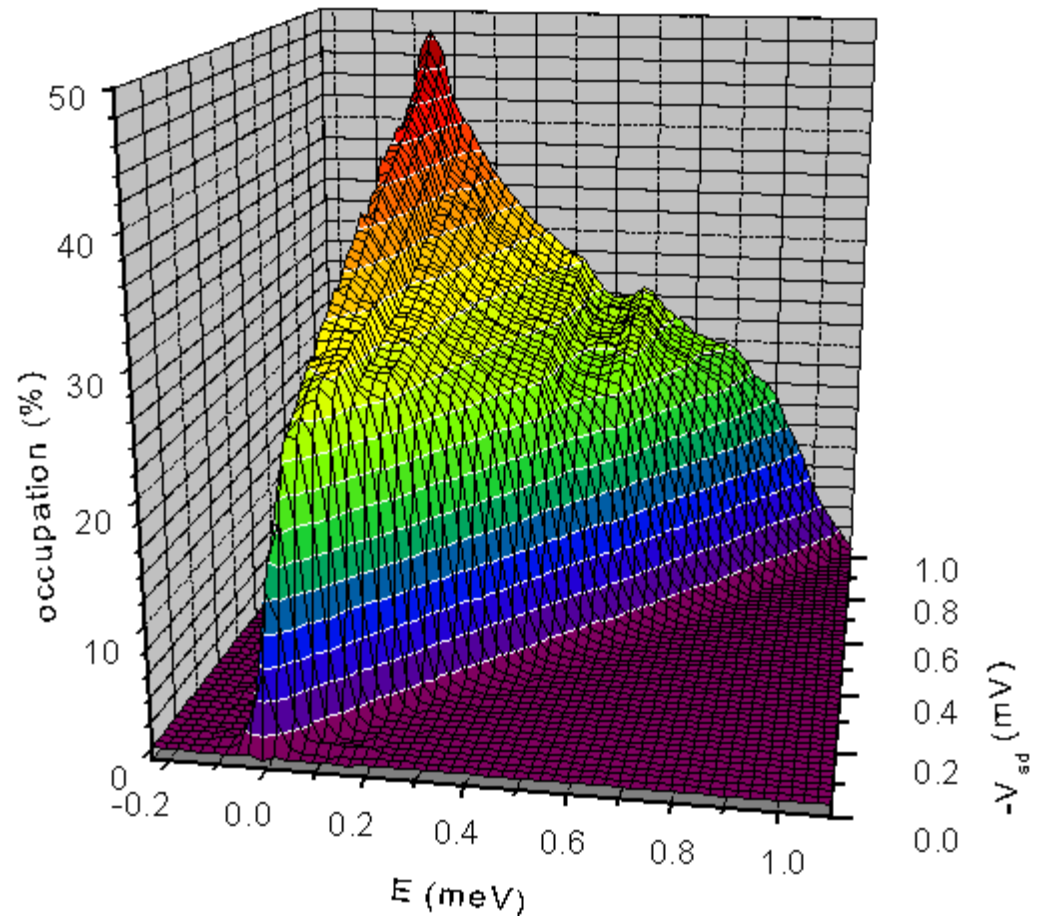
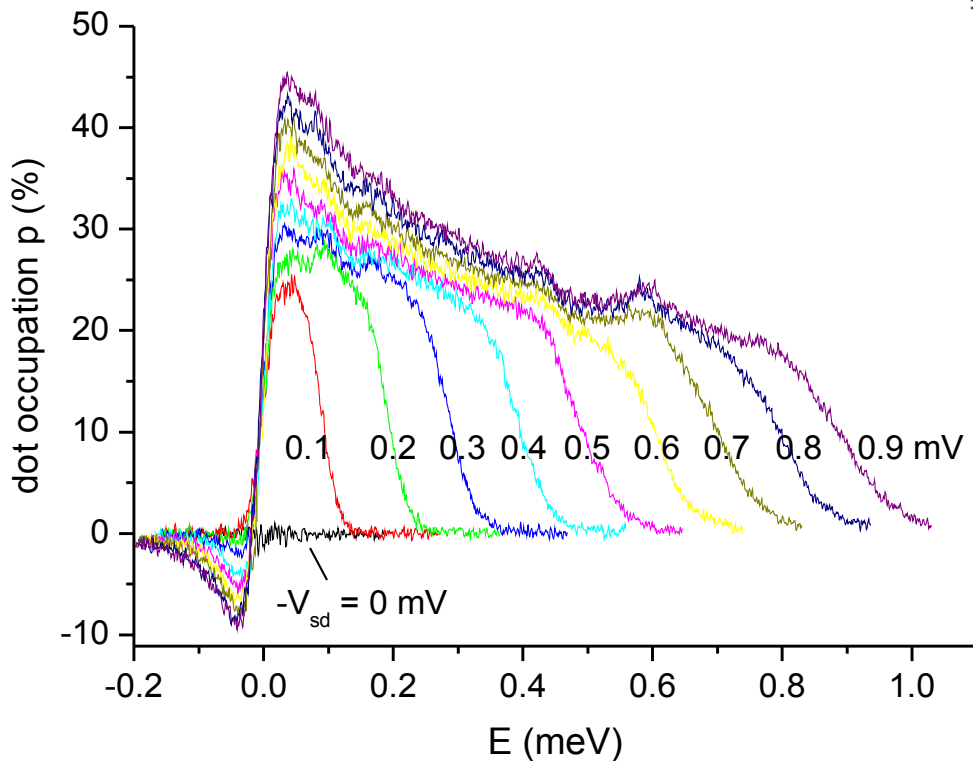


- ◆ Maximum signal shifts to larger field
  - ◆ Velocity increases with increasing energy
- ◆ Energy spread  $\Delta E$  wider for larger excess energy  $-eV$ 
  - ◆ Small energy scattering
- ◆ No noticeable change in angle spread  $\Delta B \propto \Delta\phi$ 
  - ◆ Small energy scattering results in little momentum change



# Full DOS

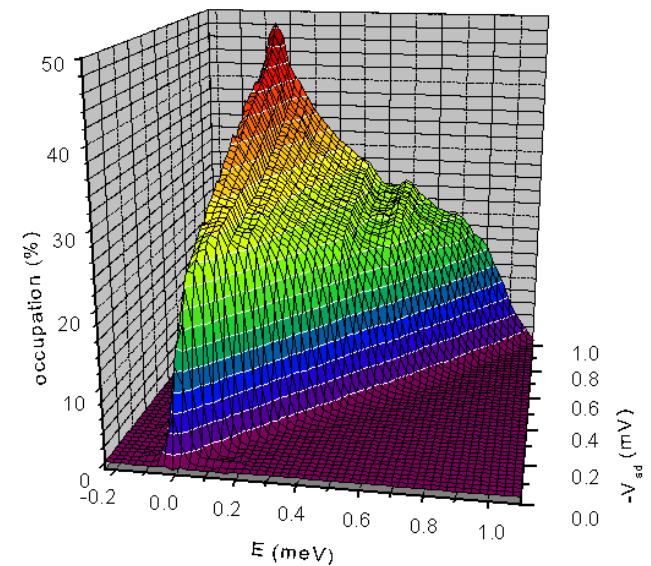
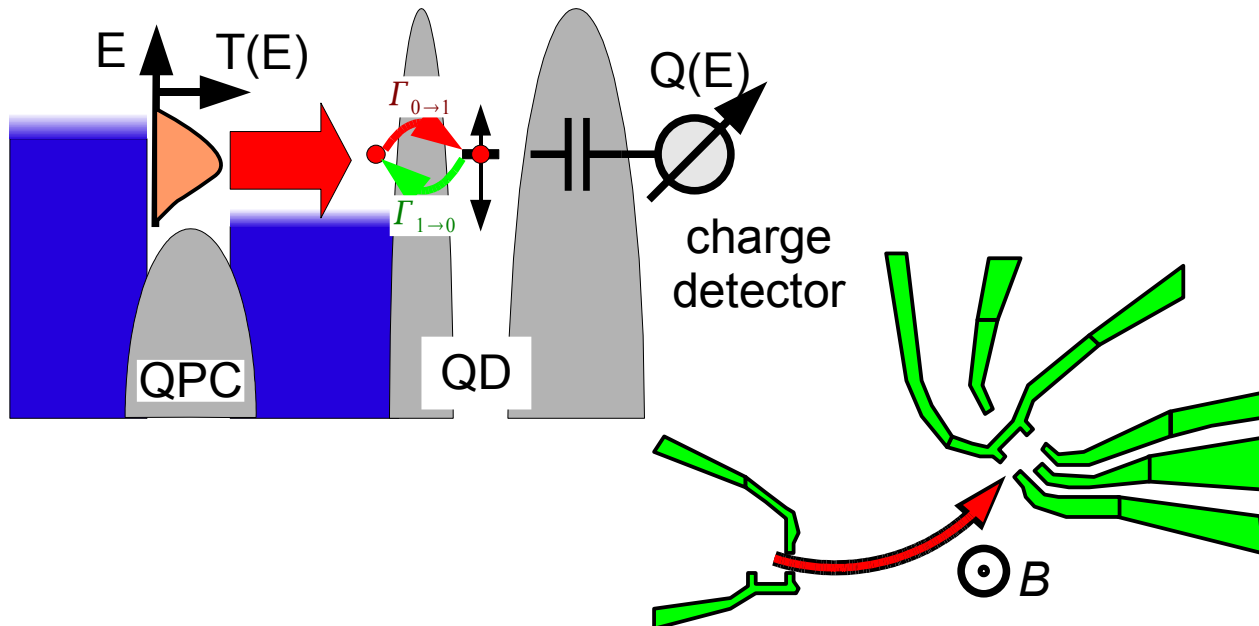
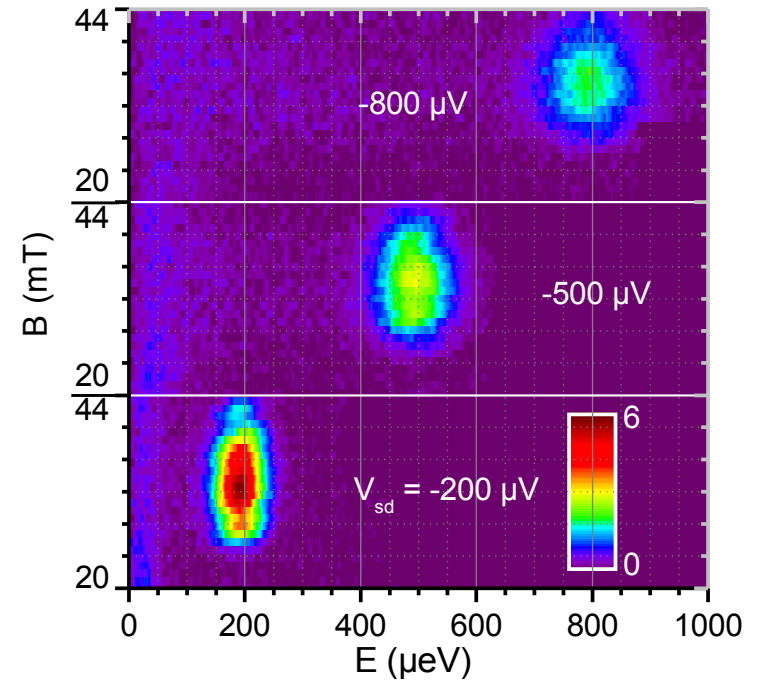
- ◆ Again QPC on 1<sup>st</sup> plateau
- ◆ Measures full energy distribution  $\Omega(E)$  of ballistic electrons



$$p = 2\Omega_{\Rightarrow} (1 - 2\Omega_{\Rightarrow} + \Omega_{\Leftarrow}) + O(\Omega^3)$$

# Summary

- ◆ Ballistic electron spectroscopy  
APL 89, 212103 (2006)
- ◆ QPC charge detector for measurement with empty dot
- ◆ Measurement of **energy and angle distribution** of non-equilibrium ballistic electrons
- ◆ Apply now: QPC (0.7 anomaly), QD (Kondo effect), energy relaxation, ....





# Thanks ...

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This work was done at the Cavendish Laboratory in Cambridge

Thanks to

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- ◆ Jonathan Griffiths and Geb A.C. Jones for the electron-beam write jobs
- ◆ Dave Ritchie for growing the wafer (himself!)

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